Simultaneous equations

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Definition 1. We call *system of equations* equations which together describe a mathematical model. A system of equations of n equations and v variables is called an $n \times v$ system or a system with $n \times v$ dimensions. If $n = v$ the system of equations is called an *exactly constrained system*, if $n < v$ an *under-constrained system*, and if $n > v$ an *over-constrained system*.

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Note 1. A *unique solution* to a system exists only if there are as many equations as variables, that is to say, if $n \geq v$. An under-constrained system may have an unlimited number of solutions or no solutions, but it may never have a unique solution. An exactly constrained or over-constrained system may have a unique solution, an infinite number of solutions, or no solution.

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Definition 2. The graph of a (2×2) linear system of equations comprise two straight lines. If the two lines intersect, then the point of intersection (x_1, y_1) satisfies both equations and therefore represents a unique solution of the system. If they do not intersect, then there are no solutions and the two corresponding equations are said to be *inconsistent* with each other. If the two equations have identical graph, then the system has an infinite number of solutions. Such equations are called *dependent* or *equivalent* equations.

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Note 2. Consider a (2×2) system of linear equations in the slope-intercept form,

$$y = m_1 x + b_1$$
$$y = m_2 x + b_2$$

if $m_1 \neq m_2$ **then**
  system has a unique solution
else
  if $b_1 \neq b_2$ **then**
    equations are inconsistent and the system has no solution
  else
    equations are equivalent and the system has infinitely many solutions

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Bibliography

Edward T Dowling. *Mathematical methods for business and economics.* Schaum's outline series, 1993